

Gravitational luminosity of a hot plasma in R^2 gravity

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Abstract The R^2 -gravity contribution to the energy loss of a hot plasma due to gravitational bremsstrahlung is calculated in the linearized theory on the basis of classical Coulomb scattering of plasma constituents in small-angle scattering approximation. The explicit dependence of the gravitational luminosity on the plasma temperature is derived and its relevance to the Einstein gravity is demonstrated. The result, when applied to the Sun as a hot plasma, shows very good agreement with available data.

1 Introduction

For a long time, the discovery of gravitational wave (GW) emissions from the compact binary system with two neutron stars PSR1913+16 [1] has been the ultimate motivation for the design, implementation, and advancement of extremely sophisticated GW detection technology. Physicists working in this field of research need this technology to conduct thorough investigations of GWs in order to advance science. The observation of GWs from a binary black hole (BH) merger (event GW150914) [2], which occurred in the 100th anniversary of Albert Einstein's prediction of GWs [3], has recently shown that this ambitious challenge has been won. The event GW150914 represented a cornerstone for science and for gravitational physics in particular. In fact, this remarkable event equipped scientists with the means to give definitive proof of the existence of GWs, the existence of BHs having mass greater than 25 solar masses and the existence of binary systems of BHs which coalesce in a time less than the age of the Universe [2]. After the event GW150914, LIGO detected a second burst of GWs from merging BHs, the event GW151226 [4]. The great hope is that such detections, also

through the collaboration with other detectors [5,6], will soon become routine and part of a nascent GW astronomy.

GW astronomy will be important for better knowledge of the Universe and also to confirm or to rule out the physical consistency of the general theory of relativity (GTR) or of any other theory of gravitation [7]. This is because, in the context of extended theories of gravity (ETG), some differences between the GTR and the other theories can be pointed out starting by the linearized theory of gravity [7]. In this picture, detectors for GWs are in principle sensitive also to a hypothetical *scalar* component of gravitational radiation, which appears in ETG like scalar-tensor gravity and $f(R)$ theories [7]. Let us clarify some important motivations which lead to a potential extension and generalization of the GTR.

Although Einstein's GTR [8] achieved great success (see for example the opinion of Landau who says that the GTR is, together with quantum field theory, the best scientific theory of all [9]) and withstood many experimental tests, it also displayed many shortcomings and flaws which today make theoreticians question whether it is the definitive theory of gravity; see the reviews [10,11,42] and the references therein. As distinct from other field theories, like the electromagnetic theory, the GTR is very difficult to quantize. This fact rules out the possibility of treating gravitation like other quantum theories, and precludes the unification of gravity with other interactions. At the present time, it is not possible to realize a consistent quantum theory of gravity (QTG) which leads to the unification of gravitation with the other forces. From an historical point of view, Einstein believed that, in the path to unification of theories, quantum mechanics had to be subjected to a more general deterministic theory, which he called generalized theory of gravitation, but he did not obtain the final equations of such a theory (see for example the biography of Einstein in [12]). At present, this point of view is partially retrieved by some theorists, starting from the Nobel Laureate 't Hooft [13].

However, one has to recall that, during the last 30 years, a strong, critical discussion of both GTR and quantum mechan-

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ics has been going on by theoreticians in the scientific community. The first motivation for this historical discussion arises from the fact that one of the most important goals of modern physics is to obtain a theory which could, in principle, show the fundamental interactions as different forms of the same *symmetry* [10, 11, 42]. Considering this point of view, today one observes and tests the results of one or more breaks of symmetry. In this way, it is possible to say that we live in an *unsymmetrical* world. In the last 60 years, the dominant idea has been that a fundamental description of physical interactions arises from quantum field theory. In this tapestry, different states of a physical system are represented by vectors in a Hilbert space defined in a spacetime, while physical fields are represented by operators (i.e. linear transformations) on such a Hilbert space. The greatest problem is that such a quantum-mechanical framework is not consistent with gravitation, because this particular field, i.e., the metric $h_{\mu\nu}$, describes both the dynamical aspects of gravity and the spacetime background. In other words, one says that the quantization of dynamical degrees of freedom of the gravitational field is meant to give a quantum-mechanical description of the spacetime. This is an unequalled problem in the context of quantum field theories, because the other theories are founded on a fixed spacetime background, which is treated like a classical continuum. Thus, at the present time, an absolute QTG, which implies a total unification of various interactions has not been obtained. In addition, the GTR assumes a classical description of the matter which is totally inappropriate at subatomic scales, which are the scales of the relic Universe [14, 15, 42].

In the unification approaches, from an initial point of view, one assumes that the observed material fields arise from superstructures like Higgs bosons or superstrings which, undergoing phase transitions, generate actual particles. From another point of view, it is assumed that geometry (for example the Ricci curvature scalar R) interacts with material quantum fields generating back-reactions which modify the gravitational action adding interaction terms (examples are high-order terms in the Ricci scalar and/or in the Ricci tensor and non-minimal coupling between matter and gravity; see below). Various unification approaches have been suggested, but without palpable observational evidence in a laboratory environment on Earth. Instead, in cosmology, some observational evidence could be achieved with a perturbation approach [15, 42]. Starting from these considerations, one can define as ETG those semi-classical theories where the Lagrangian is modified, in respect of the standard Einstein–Hilbert gravitational Lagrangian, adding high-order terms in the curvature invariants (terms like R^2 , $R^{\alpha\beta}R_{\alpha\beta}$, $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$, $R\Box R$, $R\Box^k R$) or terms with scalar fields non-minimally coupled to geometry (terms like $\phi^2 R$); see [10, 11, 42] and the references therein. In general, one has to emphasize that terms like those are present in all the

approaches to the problem of unification between gravity and other interactions. Additionally, from a cosmological point of view, such modifications of the GTR generate inflationary frameworks which are very important as they solve many problems of the standard model of the Universe [14–16, 42].

In the general context of cosmological evidence, there are also other considerations which suggest an extension of the GTR. As a matter of fact, the accelerated expansion of the Universe, which is observed today, implies that cosmological dynamics is dominated by the so called Dark Energy, which gives a large negative pressure. This is the standard picture, in which this new ingredient is considered as a source on the right-hand side of the field equations. It should have some form of un-clustered, non-zero vacuum energy which, together with the clustered Dark Matter, drives the global dynamics. This is the so called “concordance model” (Λ CDM) which gives, in agreement with the CMBR, LSS and SNeIa data, a good picture of the observed Universe today, but presents several shortcomings such as the well-known “coincidence” and “cosmological constant” problems [17]. An alternative approach is changing the left-hand side of the field equations, to see if the observed cosmic dynamics can be achieved by extending GTR; see [7, 10, 11, 42] and the references therein. In this different context, it is not required to find candidates for Dark Energy and Dark Matter that, till now, have not been found; only the “observed” ingredients, which are curvature and baryon matter, have to be taken into account. Considering this point of view, one can think that gravity is different at various scales and there is room for alternative theories. In principle, the most popular Dark Energy and Dark Matter models can be achieved considering $f(R)$ theories of gravity, where R is the Ricci curvature [7, 10, 11, 42]. In this picture, the nascent GW astronomy could, in principle, be important. In fact, a consistent GW astronomy will be the definitive test for the GTR or, alternatively, a strong endorsement for ETG [7, 42].

According to the GTR, a system with a time varying mass moment will loss its energy by radiating the GWs [3, 9, 18]. This energy loss, at the lowest order, is proportional to the third order time derivative of the quadrupole momentum of the mass-energy distribution [18]. In R^2 -gravity, which is the simplest extension of $f(R)$ -gravity, because of the presence of third polarization mode arising from the R^2 curvature term, the situation is different: the extra massive mode contribution leads to an extra energy loss which is proportional to fourth order time derivative of the quadrupole moment [19].

By comparing the theoretical considerations with the observed decay rate of binary systems PSR B1913+16 [1] and PSR J0348+0432 [20] some constraints on the strength of the R^2 -dependent term are obtained [19, 21, 22]. In many astrophysical situations, the hot plasma of ionized atoms emits electromagnetic and gravitational radiation through the coulomb collisions between the electrons and ions [23–26].

Thus, studying the gravitational luminosity of a plasma is of general interest and may be another test for the validity of $f(R)$ theory of gravity. In [27] an expression has been derived for the amount of radiated energy in a classical gravitational bremsstrahlung in R^2 -gravity, assuming the small-angle scattering approximation. In the present paper, we apply it to derive the gravitational luminosity of a hot plasma with the gravitational bremsstrahlung as a mechanism for the energy loss. In Sect. 2, we linearize the R^2 -gravity theory and, after that, we briefly discuss the quadrupole radiation in R^2 -gravity and energy loss due to gravitational bremsstrahlung in a single Coulomb collision between two charged particles. Section 3 is devoted to the calculation of the thermal gravitational radiation of the hydrogen plasma. In Sect. 4 we finally illustrate the correction with an application to the Sun. A summary of the main results is presented in Sect. 5.

2 Linearized theory and quadrupole radiation in quadratic gravity

In the general framework of $f(R)$ gravity, the R^2 theory, which was originally proposed by Starobinski [16], has been analyzed in various interesting works; see [28–32] for example. Specifically, the non-singular behavior of this class of models is discussed in [28]. In [29] R^2 inflation is combined with the Dark Energy stage and in [30] an oscillating Universe, which is well tuned with some cosmological observations, is discussed. Finally, in [31, 32] the possibility to partially solve the Dark Matter problem in the linearized R^2 theory has been analyzed.

It is also quite important to emphasize that the R^2 is the simplest one among the class of viable models with R^m terms in addition to the Einstein–Hilbert theory. In [33], it has been shown that such models may lead to the (cosmological constant or quintessence) acceleration of the Universe as well as an early time era of inflation. Moreover, they seem to pass the Solar System tests, i.e. they have an acceptable Newtonian limit, no instabilities and no Brans–Dicke problem (decoupling of the scalar) in the scalar-tensor version.

The field equations of the R^2 -gravity can be derived from the action [28–33]

$$I = \int d^4x \sqrt{-g} (a_1 R + a_2 R^2 + 16\pi G \mathcal{L}_M), \quad (1)$$

where \mathcal{L}_M is the Lagrangian density of matter and a_2 represents the coupling constant of the R^2 term. By varying the action with respect to $g_{\mu\nu}$ one obtains

$$G_{\mu\nu} + a_2 \left(2R[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R] - 2R_{;\mu;\nu} + 2g_{\mu\nu}\square R \right) = T_{\mu\nu}^{(m)}, \quad (2)$$

with the associated Klein–Gordon equation obtained by taking the trace of Eq. (2) as

$$\square R = E^2(R + T), \quad (3)$$

where E is known as the curvature energy term and defined via $E^2 = \frac{1}{6a_2}$ [32]. Relation (3) implies the idea of considering the Ricci scalar as an effective scalar field [32].

Before starting the analysis, let us emphasize an important point. As one wants the R^2 -gravity theory to be viable, one needs that it passes the Solar System tests. Thus, one must assume that the constant coupling of the R^2 term in the gravitational action is much minor with respect to the linear term R . In this way, the variation from standard GTR is very weak and the theory can pass the Solar System tests. Regarding this important issue, there are precedent works illustrating this and we need to explicitly show that the bounds are respected. The key point is that as the effective scalar field arising from curvature is very energetic, the constant coupling of the R^2 nonlinear term $\rightarrow 0$ [34]. In this case, the Ricci curvature, which is an extra dynamical quantity in the metric formalism, must have a range longer than the size of the Solar System. An important work is Ref. [35], where it is shown that this is correct if the effective length of the scalar field l is much shorter than the value of 0.2 mm. In such a case, the presence of this effective scalar is hidden from Solar System and terrestrial experiments. Another important test concerns the deflection of light by the Sun. This effect was studied in R^2 gravity by calculating the Feynman amplitudes for photon scattering, and it was found that, to linearized order, this deflection is the same as in the standard GTR [36]. In [32] it has been shown that, in order to partially solve the Dark Matter problem, the value of the curvature energy term implies a very low value of the constant coupling of the R^2 term in the gravitational action, that is, $a_2 \simeq 10^{-34} \text{ cm}^4$ in natural units. In that case, the R^2 -gravity theory results are viable and $l \ll 0.2 \text{ mm}$ is guaranteed.

Now, let us proceed to linearize the R^2 -gravity theory. We stress that in the following linearization process we closely follow [32] with a small difference in the definition of the effective scalar field.

Starting from Eq. (3), the identifications [37]

$$\Phi \rightarrow 2a_2 R + a_1 \quad \text{and} \quad \frac{dV}{d\Phi} \rightarrow \frac{a_1 R}{3} \quad (4)$$

permit to obtain a Klein–Gordon equation for the effective scalar field Φ as

$$\square \Phi = \frac{dV}{d\Phi}. \quad (5)$$

To study GWs, one analyzes the linearized theory in vacuum with a little perturbation of the background, which is assumed

given by a Minkowskian background plus $\Phi = \Phi_0$, that is, one linearizes into a background with constant curvature [32]. One also assumes Φ_0 to be a minimum for V (natural units will be used in the linearization process):

$$V \simeq \frac{1}{2}\alpha\delta\Phi^2 \Rightarrow \frac{dV}{d\Phi} \simeq m^2\delta\Phi, \quad (6)$$

where the constant m has the dimension of mass. Setting

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \\ \Phi = \Phi_0 + \delta\Phi \quad (7)$$

to first order in $h_{\mu\nu}$ and $\delta\Phi$, one calls $\tilde{R}_{\mu\nu\rho\sigma}$, $\tilde{R}_{\mu\nu}$, and \tilde{R} the linearized quantities which correspond to $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$, and R [32]. Thus, one writes down the linearized field equations [32]:

$$\tilde{R}_{\mu\nu} - \frac{\tilde{R}}{2}\eta_{\mu\nu} = (\partial_\mu\partial_\nu h_R - \eta_{\mu\nu}\square h_R) \\ \square h_R = m^2 h_R, \quad (8)$$

with

$$h_R \equiv \frac{\delta\Phi}{\Phi_0}. \quad (9)$$

$\tilde{R}_{\mu\nu\rho\sigma}$ and Eq. (8) are invariants for gauge transformations [32],

$$h_{\mu\nu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} - \partial_{(\mu}\epsilon_{\nu)} \\ \delta\Phi \rightarrow \delta\Phi' = \delta\Phi. \quad (10)$$

Thus, one defines [32]

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu} + \eta_{\mu\nu}h_R. \quad (11)$$

Let us consider the transformation for the parameter ϵ^μ [32]

$$\square\epsilon_\nu = \partial^\mu \bar{h}_{\mu\nu}, \quad (12)$$

which permits one to choose a gauge analogous to the Lorenz one of electromagnetic waves [38]

$$\partial^\mu \bar{h}_{\mu\nu} = 0. \quad (13)$$

Now, the field equations become [32]

$$\square \bar{h}_{\mu\nu} = 0, \quad (14)$$

$$\square h_R = m^2 h_R. \quad (15)$$

The solutions of Eqs. (14) and (15) are plane waves [32],

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\vec{p}) \exp(ip^\alpha x_\alpha) + c.c. \quad (16)$$

$$h_R = a(\vec{p}) \exp(iq^\alpha x_\alpha) + c.c. \quad (17)$$

with

$$q^\alpha \equiv (\omega, \vec{p}) \quad \omega = p \equiv |\vec{p}| \\ q^\alpha \equiv (\omega_m, \vec{p}) \quad \omega_m = \sqrt{m^2 + p^2}. \quad (18)$$

Equations (14) and (16) represent the equation and the solution for the standard tensor GWs of the GTR [18]. Equations (15) and (17) are, respectively, the equation and the solution for the massive scalar mode instead [32]. We stress that the dispersion law for the modes of the massive scalar field h_R is not linear [32]. In fact, the velocity of the tensor modes $\bar{h}_{\mu\nu}$ is the light speed c , but the dispersion law (the second of Eq. (18)) for the modes of h_R is that of a massive field which is interpreted in terms of a wave packet [32]. We recall that the group-velocity of a wave packet of h_R centered in \vec{p} is [32]

$$\vec{v}_G = \frac{\vec{p}}{\omega_m}. \quad (19)$$

This is exactly the velocity of a massive particle with mass m and momentum \vec{p} . From the second of Eqs. (18) and (19) one gets

$$v_G = \frac{\sqrt{\omega_m^2 - m^2}}{\omega_m}. \quad (20)$$

As one wants a constant speed of the wave packet, one obtains [32]

$$m = \sqrt{(1 - v_G^2)}\omega_m. \quad (21)$$

Let us continue our analysis in the Lorenz gauge [38] with transformations of the type $\square\epsilon_\nu = 0$; these transformations permit us to obtain a condition of transversality for the tensor part of the field: $k^\mu A_{\mu\nu} = 0$ [32]. On the other hand, they do not give the transversality for the total field $h_{\mu\nu}$. From Eq. (11) one gets [32]

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\eta_{\mu\nu} + \eta_{\mu\nu}h_R. \quad (22)$$

At this point, in the massless case, one could set [39]

$$\square\epsilon^\mu = 0 \\ \partial_\mu\epsilon^\mu = -\frac{\bar{h}}{2} + h_R. \quad (23)$$

Equation (23) gives the total transversality of the field. On the other hand, in the massive case this is impossible [32]. In fact, if one applies the D'Alembertian operator to the second of Eq. (23) and uses the field equations (14) and (15), one gets [32]

$$\square\epsilon^\mu = m^2 h_R, \quad (24)$$

which is in contrast with the first of Eq. (23). In the same way, it is possible to show that there is no linear relation between the tensorial field $\bar{h}_{\mu\nu}$ and the massive scalar field h_R [32]. Thus, one cannot choose a gauge in which $h_{\mu\nu}$ is purely spatial (that is, one cannot set $h_{\mu 0} = 0$, see Eq. (22)) [32]. One can set the traceless condition to the field $\bar{h}_{\mu\nu}$ instead [32],

$$\square \epsilon^\mu = 0, \quad \partial_\mu \epsilon^\mu = -\frac{\bar{h}}{2}. \quad (25)$$

From Eq. (25) one gets [32]

$$\partial^\mu \bar{h}_{\mu\nu} = 0. \quad (26)$$

If one wants to preserve the conditions $\partial_\mu \bar{h}^{\mu\nu}$ and $\bar{h} = 0$ transformations like [32]

$$\square \epsilon^\mu = 0, \quad \partial_\mu \epsilon^\mu = 0, \quad (27)$$

can be used. Thus, by taking \vec{p} in the z direction, one chooses a gauge in which only A_{11} , A_{22} , and $A_{12} = A_{21}$ are different from zero [32]. Setting $\bar{h} = 0$ one gets $A_{11} = -A_{22}$. Now, one puts these equations in Eq. (22), obtaining

$$h_{\mu\nu}(t, z) = A^+(t-z)e_{\mu\nu}^{(+)} + A^\times(t-z)e_{\mu\nu}^{(\times)} + h_R(t-v_G z)\eta_{\mu\nu}. \quad (28)$$

The term $A^+(t-z)e_{\mu\nu}^{(+)} + A^\times(t-z)e_{\mu\nu}^{(\times)}$ describes the two standard tensor GW polarizations which arise from the GTR [32]. The term $h_R(t-v_G z)\eta_{\mu\nu}$ is the massive field arising from the R^2 -gravity theory instead [32]. In other words, the Ricci scalar generates a third massive GW polarization, which is not present in the standard GTR [32].

Now, the post-Newtonian expansion of the theory requires one to assume the space-time metric as a small perturbation expanded around the flat background metric. In the following we restore CGS units. After lengthy algebra one finds the energy-momentum pseudo-tensor of the gravitational field [19],

$$t_{\mu\nu} = a_1 k_\mu k_\nu \dot{h}_{\alpha\beta} \dot{h}^{\alpha\beta} - a_2 \delta_{\mu\nu} (k_\alpha k_\beta \dot{h}^{\alpha\beta})^2, \quad (29)$$

where $h_{\mu\nu}$ now denotes the fluctuating part of the space-time metric, k_μ the 4-vector tangent to the world line of a GW and $\dot{h}_{\alpha\beta} \equiv \partial_0 h_{\alpha\beta}$. The rate of energy loss of a matter system coupled to gravity is found to be [9, 18]

$$\begin{aligned} \frac{dE}{dt} &= \int_S d\sigma \hat{e}_i t^{0i} \\ &= \frac{a_1}{60} G \ddot{Q}_{ij} \ddot{Q}^{ij} - \frac{a_2}{30} G \ddot{Q}_{ij} \ddot{Q}^{ij}. \end{aligned} \quad (30)$$

The symbol \hat{e}_i stands for the unit vector along the i th axis and the quadrupole moment of mass is defined to be [9] $Q_{ij} = mx_i x_j - r^2 \delta_{ij}$. Some efforts were devoted to determine the validity of the above formula by probing the observational parameters of the binary pulsar PSR 1913+16 [19, 21, 22]. Setting $a_1 = \frac{4}{3}$ in the first term of the above equation reproduces the well-known energy loss of the GTR [18]. The gravitational energy radiated due to the Coulomb collision between an electron with charge e and speed v and an ion with charge $+Ze$ in the small-angle scattering regime can be obtained [27]:

$$\begin{aligned} |\Delta E(b)| &= G \int_{-\infty}^{\infty} dt \left[\frac{a_1}{60} \ddot{Q}_{ij}(t) \ddot{Q}^{ij}(t) - \frac{a_2}{30} G \ddot{Q}_{ij} \ddot{Q}^{ij} \right], \\ &= \frac{1}{24} \frac{Z^2 e^4 v \pi G}{b^3} a_1 - B, \end{aligned} \quad (31)$$

where $B \equiv \frac{a_2}{30} \ddot{Q}_{ij}(t) \ddot{Q}^{ij}(t)$ and b denotes the impact parameter. In the small-angle approximation, one considers the particle's trajectory as a straight line [25, 26]. Let us compute B , that is, the contribution of the R^2 term to the gravitational energy loss. For the time derivatives of the quadrupole moment one gets

$$\begin{aligned} \ddot{Q}_{ij} &= \mu [3\ddot{x}_i x_j + 9\dot{x}_i \dot{x}_j + 9\ddot{x}_i \ddot{x}_j + 3x_i \ddot{x}_j \\ &\quad - 2(3\dot{x} \cdot \ddot{x} + \dot{x} \cdot \ddot{x}) \delta_{ij}], \end{aligned} \quad (32)$$

$$\begin{aligned} \ddot{Q}_{ij} &= \mu [3\ddot{x}_i x_j + 12\dot{x}_i \dot{x}_j + 18\ddot{x}_i \ddot{x}_j + 12\dot{x}_i \ddot{x}_j + 3x_i \ddot{x}_j \\ &\quad - 2(3\dot{x} \cdot \ddot{x} + 4\dot{x} \cdot \ddot{x} + \dot{x} \cdot \ddot{x}) \delta_{ij}]. \end{aligned} \quad (33)$$

Thus, one has

$$\begin{aligned} \ddot{Q}_{ij} \ddot{Q}^{ij} &= 2m^2 (3\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} + 18\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} \\ &\quad + 18\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} + \ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} \\ &\quad + 27\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} + 9\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} - 12\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x}). \end{aligned} \quad (34)$$

Then

$$\begin{aligned} \ddot{Q}_{ij} \ddot{Q}^{ij} &= 2m^2 (3\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} + 24\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} \\ &\quad + 36\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} + 24\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} \\ &\quad + \ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} - 12\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} \\ &\quad - 16\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} + 144\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} \\ &\quad + 16\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} - 48\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} + 36\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x} \\ &\quad + 48\ddot{x} \cdot \ddot{x} \dot{x} \cdot \dot{x}). \end{aligned} \quad (35)$$

Using

$$\ddot{x} = \frac{\gamma v t}{r^3} \hat{e}_x + \frac{\gamma b}{r^3} \hat{e}_y, \quad (36)$$

$$\ddot{x} = \gamma \left(\frac{v}{r^3} - 3 \frac{v^3 t^2}{r^5} \right) \hat{e}_x - 3 \frac{\gamma v^2 t b}{r^5} \hat{e}_y, \quad (37)$$

$$\begin{aligned}\ddot{\mathbf{x}} = & \gamma \left(15 \frac{v^5 t^3}{r^7} - 9 \frac{v^3 t}{r^5} - 2 \frac{\gamma v t}{r^6} \right) \hat{e}_x \\ & + \gamma \left(15 \frac{v^4 t^2 b}{r^7} - 3 \frac{v^2 b}{r^5} - 2 \frac{\gamma b}{r^6} \right) \hat{e}_y,\end{aligned}\quad (38)$$

with $\gamma = \frac{Ze^2}{m_e}$, from (36–38) one constructs the following set of relations:

$$\ddot{\mathbf{x}} \cdot \ddot{\mathbf{x}} = 45 \frac{\gamma^2 v^8 t^4}{r^{12}} - 18 \frac{\gamma^2 v^6 t^2}{r^{10}} + 9 \frac{\gamma^2 v^4}{r^8} + O(\gamma^3), \quad (39)$$

$$\ddot{\mathbf{x}} \cdot \ddot{\mathbf{x}} = -12 \frac{\gamma^2 v^6 t^2}{r^{10}} - 2 \frac{\gamma^2 v^4 t}{r^8} + O(\gamma^3), \quad (40)$$

$$\ddot{\mathbf{x}} \cdot \ddot{\mathbf{x}} = 9 \frac{\gamma^2 v^4 t^2}{r^8} - 3 \frac{\gamma^2 v^2}{r^6} + O(\gamma^3), \quad (41)$$

$$\ddot{\mathbf{x}} \cdot \dot{\mathbf{x}} = 15 \frac{\gamma v^6 t^3}{r^7} - 9 \frac{\gamma v^4 t}{r^5} - 2 \frac{\gamma^2 v^2 t}{r^6}, \quad (42)$$

$$\ddot{\mathbf{x}} \cdot \mathbf{x} = 9 \frac{\gamma v^4 t^2}{r^5} - 2 \frac{\gamma^2}{r^4} - 3 \frac{\gamma v^2}{r^3}, \quad (43)$$

$$\ddot{\mathbf{x}} \cdot \ddot{\mathbf{x}} = 3 \frac{\gamma^2 v^4 t^2}{r^8} + \frac{\gamma^2 v^2}{r^6}, \quad (44)$$

$$\ddot{\mathbf{x}} \cdot \ddot{\mathbf{x}} = -2 \frac{\gamma^2 v^2 t}{r^6}, \quad (45)$$

$$\ddot{\mathbf{x}} \cdot \dot{\mathbf{x}} = -3 \frac{\gamma v^4 t^2}{r^5} + \frac{\gamma v^2}{r^3}. \quad (46)$$

With the help of (35), for the time derivatives of the quadrupole moment one obtains

$$\begin{aligned}\ddot{Q}_{ij} \ddot{Q}^{ij} = & 2\mu^2 \left(1224 \frac{\gamma^2 v^8 t^4}{r^{10}} - 612 \frac{\gamma^2 v^6 t^2}{r^8} + 132 \frac{\gamma^2 v^4}{r^6} \right. \\ & - 288 \frac{\gamma^2 v^8 t^3}{r^{10}} - 1080 \frac{\gamma^2 v^{10} t^6}{r^{12}} \\ & \left. - 1080 \frac{\gamma^2 v^8 t^4 b^2}{r^{12}} + 648 \frac{\gamma^2 v^6 t^2 b^2}{r^{10}} \right).\end{aligned}\quad (47)$$

Therefore, one gets the contribution of the R^2 term to the gravitational energy loss,

$$\begin{aligned}B = & G \int_{-\infty}^{\infty} \frac{a_2}{30} \ddot{Q}_{ij}(t) \ddot{Q}^{ij}(t) dt \\ = & -\frac{a_2}{15} G m^2 \int_{-\infty}^{\infty} dt \left[1224 \frac{\gamma^2 v^8 t^4}{(v^2 t^2 + b^2)^5} \right. \\ & - 612 \frac{\gamma^2 v^6 t^2}{(v^2 t^2 + b^2)^4} + 132 \frac{\gamma^2 v^4}{(v^2 t^2 + b^2)^3} \\ & + 288 \frac{\gamma^2 v^8 t^3}{(v^2 t^2 + b^2)^5} - 1080 \frac{\gamma^2 v^{10} t^6}{(v^2 t^2 + b^2)^6} \\ & \left. - 1080 \frac{\gamma^2 v^8 t^4 b^2}{(v^2 t^2 + b^2)^6} + 648 \frac{\gamma^2 v^6 t^2 b^2}{(v^2 t^2 + b^2)^5} \right] \\ = & \frac{213}{80} \frac{Z^2 e^4 v^3 \pi G}{b^5} a_2.\end{aligned}\quad (48)$$

The set of the integrals that have been used in evaluating (48) can be found in the appendix.

Thus, Eq. (31) becomes

$$\begin{aligned}|\Delta E(b)| = & G \int_{-\infty}^{\infty} dt \left[\frac{a_1}{60} \ddot{Q}_{ij}(t) \ddot{Q}^{ij}(t) - \frac{a_2}{30} G \ddot{Q}_{ij} \ddot{Q}^{ij} \right], \\ = & \frac{1}{24} \frac{Z^2 e^4 v \pi G}{b^3} a_1 - \frac{213}{80} \frac{Z^2 e^4 v^3 \pi G}{b^5} a_2.\end{aligned}\quad (49)$$

3 Thermal gravitational radiation of a hot plasma

To obtain the gravitational luminosity of a plasma with gravitational bremsstrahlung as a mechanism for the loss of its energy, one must multiply (31) with the electron flux vn_e , ion density n_i , and integrate over the impact parameter b [25,26]. Therefore, we obtain the luminosity \mathcal{L} (energy loss per volume V) of the plasma,

$$\mathcal{L} = \frac{d\mathcal{E}}{dV} = 2\pi n_i n_e v \int_{b_{\min}}^{\infty} db |\Delta E(b)| b. \quad (50)$$

This integral diverges as $b \rightarrow 0$. Thus a cut-off, denoted by b_{\min} , is introduced to get a finite result for the luminosity. Based on either classical or quantum-mechanical considerations the cut-off takes the form [25,26]

$$b_{\min} = \begin{cases} \frac{e^2}{m_e v^2}, \\ \frac{\hbar}{m_e v}, \end{cases} \quad (51)$$

respectively. The final result for the luminosity depends on which form for the cut-off is engaged. We will restrict ourselves to the hydrogen plasma. Thus, $Z = 1$ and $n_e = n_i$. Hence from (50), with the quantum-mechanical cut-off, $b_{\min} = \frac{\hbar}{m_e v}$, the energy loss takes the form

$$\mathcal{L} = 2\pi^2 \frac{e^4 n_e^2 G}{c^5} \left(\frac{1}{24} \frac{m_e v^3}{\hbar} a_1 - \frac{213}{240} \frac{m_e^3 v^7}{\hbar^3} a_2 \right). \quad (52)$$

One notes that the speed of light, c , is restored in (52).

By taking the thermal average of the above expression, one gets the thermal luminosity of the plasma. In many astrophysical objects, the ratio of the Coulomb interaction energy to the thermal energy is negligible, so the hot plasma behaves like an almost ideal gas [26]. Thus, one can calculate the thermal luminosity of the plasma by averaging the electron speed in (52) over a thermal distribution of speeds. For an ensemble of particles at temperature T , obeying the Maxwell–Boltzmann statistics, the thermal average is

$$\langle f(\mathbf{v}) \rangle = \left(\frac{m\beta}{2\pi} \right)^{\frac{3}{2}} \int d^3 v e^{-\frac{\beta}{2} m v^2} f(\mathbf{v}), \quad \beta = \frac{1}{k_B T}, \quad (53)$$

where $f(\mathbf{v})$ is an arbitrary function of particle's velocity and k_B denotes the Boltzmann constant. In particular, one obtains

$$\langle v^{2n+1} \rangle = \frac{2}{\pi^{n+1}} \left(\frac{2\pi}{\beta m} \right)^{n+\frac{1}{2}} (n+1)! \quad (0 \leq n), \quad (54)$$

$$\langle v^{2n} \rangle = \frac{(2n+1)!!}{\beta^n m^n}. \quad (55)$$

Thus, from (52) and (54) we obtain the gravitational luminosity with the quantum-mechanical cut-off,

$$\begin{aligned} \langle \mathcal{L} \rangle &= 2\pi^2 \frac{e^4 n_e^2 G}{c^5} \left(\frac{1}{12} \frac{m_e a_1}{\hbar} \langle v^3 \rangle - \frac{213}{120} \frac{m_e^3 a_2}{\hbar^3} \langle v^7 \rangle \right) \\ &= \sqrt{2\pi^3} \frac{e^4 n_e^2 G}{c^5} \left[\frac{4}{6} \frac{m_e a_1}{\hbar} \left(\frac{k_B T}{m_e} \right)^{\frac{3}{2}} - \frac{3408}{5} \frac{m_e^3 a_2}{\hbar^3} \left(\frac{k_B T}{m_e} \right)^{\frac{7}{2}} \right]. \end{aligned} \quad (56)$$

Now, it is evident how the presence of the $a_2 R^2$ term in the action affects the gravitational luminosity of a hot plasma. Equation (56) stands as our final result for the gravitational luminosity. Setting $a_2 = 0$ in (56) yields

$$\langle \mathcal{L} \rangle \sim \frac{e^4 n_e^2 G m_e}{c^5 \hbar} \left(\frac{k_B T}{m_e} \right)^{\frac{3}{2}}, \quad (57)$$

which is the well-known result derived earlier by Weinberg within the context of the GTR [23].

4 Gravitational luminosity of the Sun

For an astrophysical application we use Eq. (56) to calculate the gravitational energy loss of the Sun within the framework of the R^2 -gravity theory. The total gravitational luminosity of the Sun is $L_\odot = V_\odot \langle \mathcal{L}_\odot \rangle$ where V_\odot denotes the Sun's volume. By setting $a_1 = \frac{4}{3}$, Eq. (56) takes the form

$$\begin{aligned} L_\odot &= L_\odot^{(1)} + L_\odot^{(2)} \\ &= \left[\sqrt{2\pi^3} \frac{8}{9} \frac{m_e e^4 n_e^2 G}{\hbar c^5} \left(\frac{k_B T}{m_e} \right)^{\frac{3}{2}} \right. \\ &\quad \left. - \frac{3408}{5} \sqrt{2\pi^3} a_2 \frac{m_e^3 e^4 n_e^2 G}{c^5 \hbar^3} \left(\frac{k_B T}{m_e} \right)^{\frac{7}{2}} \right] V_\odot. \end{aligned} \quad (58)$$

Based on the massive scalar mode arising from the R^2 term, the coupling constant a_2 comes to be very small with respect to linear term R . Assuming the typical galactic scale for the curvature energy, $E \simeq 10^{45}$ g, we find $a_2 = 10^{-34}$ cm⁴ in natural units [32]. In this way, the variation from the standard GTR is very weak. The parameters needed to obtain the above result (in CGS units) are

$$m_e = 9 \times 10^{-28} \text{ g}, \quad (59)$$

$$e = 4.8 \times 10^{-10} \text{ esu} \quad (60)$$

$$G = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}, \quad (61)$$

$$\hbar = 10^{-27} \text{ erg s}, \quad (62)$$

$$k_B = 1.38 \times 10^{-16} \text{ erg K}^{-1}, \quad (63)$$

$$c = 3 \times 10^{10} \text{ cm s}^{-1}, \quad (64)$$

$$n_e = 3 \times 10^{25} \text{ cm}^{-3}, \quad (65)$$

$$V_\odot = 2 \times 10^{31} \text{ cm}^3, \quad (66)$$

$$T_\odot = 10^7 \text{ K}. \quad (67)$$

Therefore one straightforwardly calculates

$$L_\odot^{(1)} \simeq 10^{16} \text{ erg s}^{-1}, \quad (68)$$

$$L_\odot^{(2)} \simeq 10^{13} \text{ erg s}^{-1}. \quad (69)$$

As we can see, the first term in Eq. (68), that coming from standard GTR, is in good agreement with available results [23, 24, 40]. It is important to clarify the physical reason of the very small contribution of the R^2 term in Eq. (69). The reader could indeed think that we did not take into account all effects of the R^2 gravity theory under investigation or that we did not correctly choose the PPN-restricted parameters for the action (1). In fact, there may be GWs enhanced from some models of $f(R)$ gravity on the order of 15% [41]. The key point here is that, exactly in order to match the PPN-restricted parameters for the action (1) and to be consistent with solar system tests, we have set the coupling constant of the R^2 term to be very small. Such a setting has been chosen also to match the Dark Matter model in [31, 32]. This crucial point makes the contribution of Eq. (69) small.

5 Concluding remarks

A new era in astrophysics and gravitation started with the events GW150914 [2] and GW151226 [4]. In fact, on the one hand the nascent GW astronomy will be important for a better knowledge of the Universe. On the other hand, it will permit one to confirm or to rule out the physical consistency of the GTR or of any other theory of gravitation [7]. A key point is indeed that, in the framework of the ETG, some differences between the GTR and the other theories can be pointed out starting by the linearized theory of gravity [7]. Some important motivations which lead to a potential extension and generalization of GTR have been stressed in the Introduction of this paper. The most important issue is, perhaps, the possibility to see the ETG as a potential alternative to Dark Matter and Dark Energy [7, 10, 11]. Considering this different approach, gravity could be different at different scales and there is room for alternative theories. In fact, Dark Energy and Dark Matter can be, in principle, achieved if one considers $f(R)$ theories of gravity, where R is the Ricci curvature [7, 10, 11]. In this alternative framework, the nascent GW astronomy should be important because a con-

sistent GW astronomy will be the definitive test for the GTR or, alternatively, a strong endorsement for ETG [7].

In GTR, a system with a time varying mass moment will lose its energy by radiating gravitational radiation [3, 9, 18]. At the lowest order the energy loss is proportional to the third order time derivative of the quadrupole momentum of the mass-energy distribution [18]. R^2 -gravity represents the simplest extension of $f(R)$ -gravity. It shows the presence of a third polarization mode arising from the R^2 curvature term. In that case, the situation is different. In fact, the extra massive mode contribution leads to an extra energy loss, which is proportional to the fourth order time derivative of the quadrupole moment [19]. If one compares the theoretical considerations with the observed decay rate of binary systems PSR B1913+16 [1] and PSR J0348+0432 [20], one can obtain some constraints on the strength of the R^2 -dependent term [19, 21, 22]. There are many astrophysical situations where the hot plasma of ionized atoms emits both electromagnetic waves and GWs through the Coulomb collisions between the electrons and ions [23–26]. Hence, the analysis of the gravitational luminosity of a plasma should be of general interest and may be, in principle, another test for the validity of $f(R)$ theories of gravity. An expression for the amount of radiated energy in a classical gravitational bremsstrahlung in R^2 -gravity has been derived in [27] through the assumption of the small-angle scattering approximation. In this paper, we applied it to a derivation of the gravitational luminosity of a hot plasma with gravitational bremsstrahlung as a mechanism for the energy loss. After linearizing the R^2 -gravity theory, we briefly discussed the quadrupole radiation in R^2 -gravity and the energy loss due to gravitational bremsstrahlung in a single Coulomb collision between two charged particles. Then we calculated the thermal gravitational radiation of the hydrogen plasma. Finally, we illustrated the correction with an application to the Sun. The presence of the massive term in Eq. (28) is a characteristic of higher-order terms in $f(R)$ -gravity. Thus, R^2 -gravity theory includes massive GW modes. Hence, our results, beside confirming the standard GTR, stimulate the validity of $f(R)$ -gravity. Until now there is not available data to confront our result to the experiment and fix the parameter of the R^2 -gravity contribution. We also stress the possibility to generalize the calculations in this paper for other modified gravity theories, listed in [11].

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Appendix: computation of some elementary integrals

The following set of integrals have been used in evaluating (48):

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{x^2 dx}{(a^2 x^2 + b^2)^3} &= \frac{\pi}{8a^3 b^3}, \\ \int_{-\infty}^{\infty} \frac{dx}{(a^2 x^2 + b^2)^2} &= \frac{\pi}{2ab^3}, \\ \int_{-\infty}^{\infty} \frac{x^4 dx}{(a^2 x^2 + b^2)^5} &= \frac{3\pi}{128a^5 b^5}, \\ \int_{-\infty}^{\infty} \frac{x^2 dx}{(a^2 x^2 + b^2)^{\frac{9}{2}}} &= \frac{16}{105a^3 b^6}, \\ \int_{-\infty}^{\infty} \frac{x^2 dx}{(a^2 x^2 + b^2)^4} &= \frac{\pi}{16a^3 b^5}, \\ \int_{-\infty}^{\infty} \frac{dx}{(a^2 x^2 + b^2)^4} &= \frac{5\pi}{16ab^7}, \\ \int_{-\infty}^{\infty} \frac{dx}{(a^2 x^2 + b^2)^{\frac{7}{2}}} &= \frac{16}{15ab^6}, \\ \int_{-\infty}^{\infty} \frac{x^6 dx}{(a^2 x^2 + b^2)^6} &= \frac{3\pi}{256a^7 b^5}, \\ \int_{-\infty}^{\infty} \frac{x^4 dx}{(a^2 x^2 + b^2)^6} &= \frac{3\pi}{256a^5 b^7}, \\ \int_{-\infty}^{\infty} \frac{x^2 dx}{(a^2 x^2 + b^2)^5} &= \frac{5\pi}{128a^3 b^7}, \\ \int_{-\infty}^{\infty} \frac{dx}{(a^2 x^2 + b^2)^3} &= \frac{3\pi}{8ab^5}. \end{aligned}$$

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